

Theorem :- If A and B are independent events then

- (i) A and B' are independent events
- (ii) A' and B are independent events
- (iii) A' and B' are independent events

Proof

Let A and B are independent events

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B) \quad (1)$$

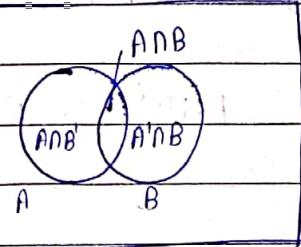
$$\text{or } P(A \cap B) = P(A) - P(A \cap B')$$

i) $A = (A \cap B') \cup (A \cap B)$

Since $(A \cap B')$ and $(A \cap B)$

are mutually disjoint

$$\text{then } P(A) = P(A \cap B') + P(A \cap B)$$



$$\Rightarrow P(A) = P(A \cap B') + P(A) \cdot P(B)$$

$$\Rightarrow P(A \cap B') = P(A) - P(A)P(B)$$

$$= [P(A) \{ 1 - P(B) \}]$$

$$= P(A) \cdot P(B')$$

So A and B' are independent events.

ii) $B = (A \cap B) \cup (A' \cap B)$

$$P(B) = P(A \cap B) + P(A' \cap B) \quad \{ \text{since } (A \cap B) \text{ and } (A' \cap B) \text{ are mutually exclusive} \}$$

$$P(B) = P(A) \cdot P(B) + P(A') \cdot P(B)$$

$$\Rightarrow P(A' \cap B) = P(B) - P(A)P(B)$$

$$= P(B) \{ 1 - P(A) \}$$

$$= P(B)P(A')$$

So A' and B are independent events.

Then iii) $P(A' \cap B') = P(A \cup B)' \Rightarrow 1 - P(A \cup B)$

and it is true.

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Then, $P(A \cup B)' = P(A)' - P(B)' + P(A)' \cdot P(B)'$

Now, $[1 - P(A)] - [1 - P(B)] + [1 - P(A)] \cdot [1 - P(B)]$

$$= [1 - P(A)] [1 - P(B)]$$

$$= P(A') P(B')$$

So, A' and B' are independent events.

a) $P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|AB)$

$$\begin{aligned} \text{L.H.S. } P(A \cap B \cap C) &= P(AB) \cdot P(C|AB) \\ &= P(A) \cdot P(B|A) \cdot P(C|AB) \end{aligned}$$

Q) Two dice are thrown. Find the prob. that the sum of the no. coming up on them is 9 if it is known that the no. 5 always occurs on 1st die.

Soln: Let E be the event of getting 5 on first die.

F be the event of getting sum 9.

Now, $P(E) = \frac{1}{6}$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

a) Two coins are tossed. What is the prob. of coming up of two heads if it is known that at least one head comes up.

Soln Let A is the event of getting two head.
B is the event of getting atleast one head.

$$S = \{HH, HT, TH, TT\}$$

$$\begin{aligned} P(B) &= \frac{3}{4}, \quad P(A|B) = \frac{P(A \cap B)}{P(B)} \\ &= \frac{1/4}{3/4} = \frac{1}{3} \end{aligned}$$

Theorem :- If A and B are two events such that $B \neq \emptyset$ then

$$P(A|B) + P(A'|B) = 1$$

~~proof~~

$$\begin{aligned} &P(A|B) + P(A'|B) \\ &= \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} \end{aligned}$$

$$= \frac{P(A \cap B) + P(A' \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)} = 1$$

(*) A bag contains 4 white and 2 black balls. Another bag contains 3 white and 5 black balls. If one ball is drawn from each bag find the prob. that

- (i) Both are white
- (ii) Both are black
- (iii) One is white, one is black

~~Sol~~ ① Let E be the event of getting a white ball from the first bag and F be the event of getting a white ball from the 2nd bag. Then

$$P(E) = \frac{4}{6}, P(F) = \frac{3}{8}$$

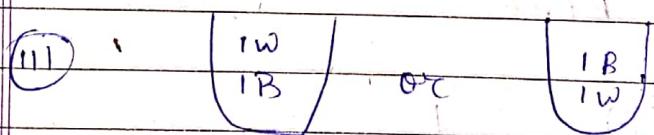
$$P(E \cap F) = P(E) \cdot P(F)$$

$$= \frac{4}{6} \cdot \frac{3}{8} = \frac{1}{4}$$

② Let A be the event of getting a black ball from 1st bag. B be the event of getting black ball from 2nd bag.

Now $P(A) = \frac{2}{6}, P(B) = \frac{5}{8}$

$$\Rightarrow P(A \cap B) = \frac{2}{6} \times \frac{5}{8} = \frac{5}{24}$$



$$E_1 = \frac{4}{6} \times \frac{5}{8} + \frac{2}{6} \times \frac{3}{8}$$

and if it is a single pack then

then total = $\frac{2}{6} + \frac{5}{8} = \frac{13}{24}$

now if it is 1.48 weight. $\frac{24}{24}$ and so

total being half and one

③ If A and B are not impossible events then they can not be mutually exclusive as well as independent events.

~~Sol~~ Let A and B are mutually exclusive

events A and B and $P(A) > 0$ and $P(B) > 0$ — (1)

also $P(A \cap B) = P(A) \cdot P(B/A) = P(A) \cdot P(A/B)$ — (2)

that all events A and B are independent (By multiplication theorem)

$\Rightarrow P(A/B) = 0 + P(A)$ from (1)

$$\Rightarrow P(A/B) \neq P(A)$$

$$\text{and } P(B/A) = 0 + P(B)$$

$$\Rightarrow P(B/A) \neq P(B)$$

$\therefore A$ and B are not independent.

Now let A and B are independent events

and $P(A) > 0$ & $P(B) > 0$

$$P(A \cap B) = P(A) \cdot P(B) \neq 0$$

$\therefore A$ and B are not mutually exclusive events.

- (Q) A bag contains 4 white and 3 red balls. Two draws of one ball each are made without replacement. What is the prob. that both the balls are red.

Sol) Let A be the event of getting red ball in first draw and B be the event of getting red ball in second draw.

$$P(A \cdot B) = P(A) \cdot P(B/A)$$

$$= \frac{3C_1}{7C_1} \times \frac{2C_1}{6C_1} = \frac{1}{7}$$

- (a) A coin is thrown twice. Let the event E be the first throw's result is a head and the event F be the last throw result is a tail. Find whether the events E and F are independent.

Soln Let E be the event that first throw results in a head. F be the event that the last throw results in a tail.

$$E = \{HTT, HHT, HHH, HTTH\}$$

$$F = \{TTT, HTT, HHT, THT\}$$

$$P(E \cap F) = P(E) \cdot P(F)$$

L.H.S

$$P(E \cap F) = \frac{2}{8} = \frac{1}{4}$$

$$\text{R.H.S } P(E) \cdot P(F) = \frac{4}{8} \cdot \frac{4}{8} = \frac{1}{4}$$

$$\text{Here L.H.S} = \text{R.H.S}$$

Hence E and F are independent events.

- (b) If the events A and B are such that $P(A) \neq 0$, $P(B) \neq 0$ and A is independent of B then B is independent of A.

Soln Here A is independent to B

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

Now

$$\frac{P(B|A)}{P(A)} = P(B)$$

$$\Rightarrow P(A) = P(A|B)$$

$\therefore B$ is independent of A .

Bayes' Theorem -

Bayes' theorem is based on the concept that the prob. should be revised when some new information is available. The idea of revising the prob. is used by all of us in daily life. Even though we may not be knowing any thing prob.

Ex :- A student while going to college may start without a rain coat but as soon as he comes out of his home and sees a thick mass of clouds in the sky he may decide to take a rain coat with him. Thus he revised his earlier decision of going to college without a rain coat.

In the same way the probabilities are revised as soon as new information is available about the problem concerned. The necessity of revising probabilities arises from a need to make better use of available information and thereby reduce the element of risk involved in decision making. The idea of revising prob. on the basis of new information was given by british mathematician Thomas Bayes.

Bayes theorem offers a powerful statistical method of evaluating new information and revising our prior estimate (Based on the limited information) of the probability. Modern decision theory is also called Bayesian decision theory.

~~Bayes theorem -~~

If A_1, A_2, \dots, A_n are mutually exclusive and exhaustive events and A is an event which occurs together (in conjunction) with either of A_i i.e. if A_1, A_2, \dots, A_n form a partition of the sample space S and A be any event then

$$P(A_k/A) = \frac{P(A_k) \cdot P(A/A_k)}{P(A_1) \cdot P(A/A_1) + P(A_2) \cdot P(A/A_2) + \dots + P(A_n) \cdot P(A/A_n)}$$

A_1	A_2	\dots	A_k	\dots	A_n
\cap			\cap		

Bayes theorem can be used only when it is known that an event A has occurred and we have to find the prob. of the occurrence of other event A_k such that events $A_1, A_2, \dots, A_k, \dots, A_n$ are mutually exclusive and exhaustive i.e. they are mutually exclusive and cover all possible cases of an experiment and A is known to occur with one of A_i .

~~Proof~~: Since A_1, A_2, \dots, A_n from a partition of sample space S therefore

i) A_1, A_2, \dots, A_n are non empty.

ii) $A_i \cap A_j = \emptyset$ where $i \neq j$

iii) $S = A_1 \cup A_2 \cup A_3 \dots \cup A_k \cup \dots \cup A_n$

Now $A_i = A \cap S_i$ and S_i is disjoint

and $S = A \cap (A_1 \cup A_2 \cup A_3 \dots \cup A_n)$

$\therefore P(A) = (A \cap A_1) \cup (A \cap A_2) \cup (A \cap A_3) \dots \cup (A \cap A_n) \quad \text{--- (1)}$

Since A_1, A_2, \dots, A_n are disjoint sets then

$A \cap A_1, A \cap A_2, A \cap A_3, \dots, A \cap A_n$ are also

disjoint sets hence disjoint.

Now from (1) by addition theorem

$$P(A) = P(A \cap A_1) + P(A \cap A_2) + \dots + P(A \cap A_n)$$

$$= P(A_1) P(A/A_1) + P(A_2) P(A/A_2) + \dots + P(A_n) P(A/A_n) \quad \text{--- (1)}$$

Now,

$$P(A_k/A) = \frac{P(A_k \cap A)}{P(A)}$$

$$P(A_k/A) = \frac{P(A/A_k) \cdot P(A_k)}{P(A_1) P(A/A_1) + P(A_2) P(A/A_2) + \dots + P(A_n) P(A/A_n)}$$

NOTE :-

If $P(A_1) = P(A_2) = P(A_3) = \dots = P(A_n)$ then

$$P(A_k/A) = \frac{P(A/A_k)}{P(A/A_1) + P(A/A_2) + \dots + P(A/A_n)}$$

* The probabilities $p(A_1), p(A_2), \dots, p(A_n)$ which are known before the experiment takes place are called priori prob. and $p(A_i|A)$ are called updated probabilities or posteriori prob.

Q) Suppose a desk has two drawers. Drawer 1 contains 3 green and 2 white balls. Drawer 2 contains 4 green and 3 white balls, all the balls differ only with respect to colour. A drawer is picked at random a ball is drawn from it. What is the prob. that this ball is green? What is prob. of drawing green ball from drawer 2?

Soln Let A be the event of getting a green ball.
 B_1 be the event of getting ball from drawer 1.

B_2 be the event of getting ball from drawer 2.

$$\begin{aligned} P(A) &= P(A|B_1) + P(A|B_2) \\ &= P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2) \\ &= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{4}{7} = \frac{41}{70} \end{aligned}$$

$$\begin{aligned} P(B_2|A) &= \frac{P(A|B_2) \cdot P(B_2)}{P(A)} \\ &= \frac{\frac{1}{2} \cdot \frac{4}{7}}{\frac{41}{70}} = \frac{20}{41} \end{aligned}$$

Q) A man is known to speak the truth 3 out of 4 times. A throws a die and reports that it is a 6. Find the prob. that it is actually a six.

Soln Let A be event man speaks.

A_1 be event that it actually 6 occurs.

A_2 be event that it actually not occurs 6.

$$P(A_1) = \frac{1}{6}, P(A_2) = \frac{5}{6}$$

$$P(A|A_1) = \frac{3}{4}, P(A|A_2) = \frac{1}{4}$$

$$P(A_1|A) = \frac{P(A|A_1) \cdot P(A_1)}{P(A)}$$

$$= \frac{\frac{3}{4} \cdot \frac{1}{6}}{\frac{1}{3}} = \frac{3}{8}$$

$$P(A) = P(A_1) \cdot P(A|A_1) + P(A_2) \cdot P(A|A_2)$$

$$= \frac{1}{6} \cdot \frac{3}{4} + \frac{5}{6} \cdot \frac{1}{4}$$

$$= \frac{3+5}{24} = \frac{1}{3}$$

Q) The contents of urns A, B, C are as follows:

(A) 1 white, 2 black and 3 red balls.

(B) 2 white, 1 black and 1 red ball

(C) 4 white, 5 black and 3 red balls

One urn is chosen at random and two balls are drawn. They happen to

Two balls are drawn from a bag containing 3 white and 2 red balls. What is the probability that they come from (A, B) or (C)?

Let E_1 be the event that the two balls are drawn from A.

E_2 be the event that the two balls are drawn from B.

E_3 be the event that the two balls are drawn from C.

E be the event that two balls are taken from the selected even wins are white and red.

Now,

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P\left(\frac{E}{E_1}\right) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1 \times 3}{6 \times 5} = \frac{1}{5}$$

$$P\left(\frac{E}{E_2}\right) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2 \times 1}{4 \times 3} = \frac{1}{6}$$

$$P\left(\frac{E}{E_3}\right) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{4 \times 3}{12 \times 11} = \frac{2}{11}$$

$$\text{Now } P\left(\frac{E_1}{E}\right) = \frac{P(E_1) P\left(\frac{E}{E_1}\right)}{P(E_1) P\left(\frac{E}{E_1}\right) + P(E_2) P\left(\frac{E}{E_2}\right) + P(E_3) P\left(\frac{E}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{2}{11}}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{2}{11}} = \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$$

$$\begin{aligned}
 \text{Now } P(E_2/E) &= \frac{P(E_2) \cdot P\left(\frac{E}{E_2}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right)} \\
 &= \frac{\frac{1}{3} \cdot \frac{1}{3}}{\frac{1}{3} \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{11}} \\
 &= \frac{55}{118}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now } P\left(\frac{E_3}{E}\right) &= \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right) + P(E_3)P\left(\frac{E}{E_3}\right)} \\
 &= \frac{\frac{1}{3} \cdot \frac{2}{11}}{\frac{1}{3} \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{11}} \\
 &= \frac{30}{118} \quad \underline{\text{deg}}
 \end{aligned}$$

(Q) A card from a pack of 52 cards is lost from the remaining card of the pack of two cards are drawn and are found to be spades. Find the probability of the lost card being a spade.

Soln Let E_1, E_2, E_3, E_4 be the events of losing a card of spades, clubs, hearts and diamonds respectively then

$$P(E_i) = P(E_2) = P(E_3) = P(E_4) = \frac{13C_1}{52C_1} = \frac{1}{4}$$

Now let E be the event of drawing two spades from the remaining 51 cards.

Now $P\left(\frac{E}{E_1}\right)$ = Prob. of drawing two spades given that a card of spade is missing

$$= \frac{^{12}C_2}{^{51}C_2}$$

$P\left(\frac{E}{E_2}\right)$ = Prob. of drawing two spades given that a card of club is missing

$$\frac{^{13}C_2}{^{51}C_2}$$

$$\text{Similarly } P\left(\frac{E}{E_3}\right) = P\left(\frac{E}{E_4}\right) = \frac{^{13}C_2}{^{52}C_2}$$

$$\text{Now } P\left(\frac{E_1}{E}\right) = P(E_1) \cdot P\left(\frac{E}{E_1}\right)$$

$$P(E_1) P\left(\frac{E}{E_1}\right) + P(E_2) P\left(\frac{E}{E_2}\right) + P(E_3) P\left(\frac{E}{E_3}\right) + P(E_4) P\left(\frac{E}{E_4}\right)$$

$$P(E_1) + P\left(\frac{E}{E_1}\right)$$

$$\frac{1}{4} \cdot \frac{^{12}C_2}{^{51}C_2}$$

$$\frac{1}{4} \cdot \frac{^{12}C_2}{^{51}C_2} + \frac{1}{4} \cdot \frac{^{13}C_2}{^{51}C_2} + \frac{1}{4} \cdot \frac{^{13}C_2}{^{51}C_2} + \frac{1}{4} \cdot \frac{^{13}C_2}{^{51}C_2}$$

$$\frac{^{12}C_2}{^{51}C_2}$$

$$\frac{^{12}C_2 + ^{13}C_2 + ^{13}C_2 + ^{13}C_2}{^{51}C_2}$$

$$= \frac{^{12}C_2}{^{51}C_2}$$

$$= \frac{12 \times 11}{51 \times 50}$$

(a) In an examination an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The prob. that he makes a guess is $\frac{1}{3}$ and the prob. that he copies the answer is $\frac{1}{6}$. The prob. that his answer is correct given that he copied it is $\frac{1}{8}$. The prob. that his answer is correct given that he guessed it is $\frac{1}{4}$. Find the prob. that he knows the answer to the question given that he correctly answered it.

Soln: Let E_1, E_2, E_3 be the events that an examinee guesses, copies and knew the answer to the question.
Let E be the event that he correctly answered.

$$P(E_1) = \frac{1}{3}, \quad P(E_2) = \frac{1}{6}, \quad P(E_3) = 1 - \frac{1}{3} - \frac{1}{6} \\ = \frac{1}{2}$$

$$\text{Now: } P(E|E_1) = \frac{1}{4}$$

$$P(E|E_2) = \frac{1}{8}$$

$$P(E|E_3) = 1$$

Now,

$$P\left(\frac{E_3}{E}\right) = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right)}$$

Ex. If the value of $\frac{1}{2}$ is with probability $\frac{1}{4}$, $\frac{1}{3}$ with probability $\frac{1}{6}$, $\frac{1}{8}$ with probability $\frac{1}{12}$ and $\frac{1}{2}$ with probability $\frac{1}{2}$ then the random variable x is defined by $x = \begin{cases} \frac{1}{2} & \text{with prob. } \frac{1}{4} \\ \frac{1}{3} & \text{with prob. } \frac{1}{6} \\ \frac{1}{8} & \text{with prob. } \frac{1}{12} \\ \frac{1}{2} & \text{with prob. } \frac{1}{2} \end{cases}$

RANDOM VARIABLE

Definition :- A real valued function which assigns a real number to each outcome in a sample space.

A random variable x associates a numerical value with each outcome of an experiment.

Ex :-

i) When we will throw a single die its possible outcomes are 6. It can be the number 1, 2, 3, 4, 5, 6 appearing on the upper most face thus in this case the random variable x takes the value 1, 2, 3, 4, 5, 6.

ii) By a random variable we mean a real number x connected with the outcome of a random experiment E for example if E consists of two tosses the random variable which is the number of heads in outcomes : HH HT TH TT
values of x : 2 1 1 0

- A random variable is also called stochastic variable or a chance variable if given to the or simply a variable. The name random variable is given to the variant x outcomes of which are uncertain and hence depends on chance.

Discrete random variable -

A variable which can assume only a countable no. of real values and for which the value that the variable takes depends on chance is called a discrete random variable (or discrete stochastic variable). In other word a real valued function on a discrete sample space is called a discrete random variable. Discrete random variable is a variable which takes all possible finite or infinite countable values over \Rightarrow sample space of random experiment.

Ex:- No. of accidents per month, no. of telephone calls per unit time, no. of successes in n trials, no. of deaths from a disease.

Continuous Random variable (2017) -

A random variable x is said to be continuous if it can take all possible values (integral as well as fractional) b/w certain limits.

Ex:- Age, Height, weight, temperature

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continuous Random variable (2017) -

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Probability Mass function (PMF) :-

If X is a random variable with distinct values $x_1, x_2, x_3, \dots, x_n$, then probability of x is defined as

$$P_X(x_i) = \begin{cases} P(X=x_i) = p_i & \text{if } x = x_i \\ 0 & \text{if } x \neq x_i ; i = 1, 2, 3, \dots \end{cases}$$

is called the prob. mass funⁿ of random variable X .

* There are two conditions

$$\textcircled{i} \quad P(X=x_i) = P(x_i) \geq 0 \forall i$$

$$\textcircled{ii} \quad \sum_{i=1}^{\infty} P(x_i) = 1$$

Let x_i is no. of heads when two coins are tossed then.

$x=x_i$	0	1	2
$P(X=x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Probability Density Function (PDF) :-

If a continuous random variable is defined with the interval $a \leq x \leq b$, then function $f(x)$ is said to be prob. density funⁿ if

$$\{ P[a \leq X \leq b] = \int_a^b f(x) dx \}$$

* Condition for p.d.f. -

(i) $\int_{-\infty}^{\infty} f(x) dx = 1$

(ii) $f(x) \geq 0$

Q) A random variable x has the following prob. fun values of $x = 0, 1, 2, 3, 4, 5, 6, 7$

$$x = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$

$$P(x) = 0 \quad k \quad 2k \quad 3k \quad 4k^2 \quad 5k^2 \quad 6k \quad 7k$$

$$P(x) = 0 \quad k \quad 2k \quad 3k \quad 3k \quad k^2 \quad 2k^2 \quad 7k^2 + k$$

i) Find k

ii) $P(x < 6)$, $P(x \geq 6)$ and $P(0 < x < 5)$

iii) If $P(x \leq a) > \frac{1}{2}$, find the min^m value of a

iv) Determine the distribution fun of x .

Soln

i) we know that

$$\sum_{x_i=0} P(x = x_i) = 1$$

$$\Rightarrow \sum_{x_i=0} P(x = x_i) = 1$$

$$\Rightarrow 0 + k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 9k + 10k^2 = 1 \Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow 10k^2 + 10k - k - 1 = 0$$

$$\Rightarrow 10k(k+1) - 1(k+1) = 0$$

$$\Rightarrow (10k-1)(k+1) = 0$$

$$\Rightarrow k = \frac{1}{10} \text{ or } k = -1$$

But prob. cannot be -ve.

$$\therefore k = \frac{1}{10}$$

$$\begin{aligned}
 \text{ii) } P(X < 6) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\
 &\quad + P(X=4) + P(X=5) \\
 &= 0 + K + 2K + 2K + 3K + K^2 \\
 &= 8K + K^2 \\
 &= \frac{8}{10} + \frac{1}{100}
 \end{aligned}$$

$$\begin{aligned}
 \text{Required value} &= 8K + \frac{1}{100} \\
 &= 8 \times \frac{1}{10} + \frac{1}{100} \\
 &= \frac{8}{10} + \frac{1}{100} \\
 P(X \geq 6) &= 1 - P(X < 6) \\
 &= 1 - \frac{8}{10} - \frac{1}{100} = \frac{19}{100}
 \end{aligned}$$

$$\begin{aligned}
 P(0 \leq X \leq 5) &= P(X=1) + P(X=2) + P(X=3) + P(X=4) \\
 &= 8K + 2K + 2K + 3K \\
 &= 8K \\
 &= \frac{8}{10} = \frac{4}{5}
 \end{aligned}$$

v.v.i

iv)	X	$F_X(x) = P(X \leq x)$
	0	$0 = 0$
	1	$K = \frac{1}{10}$
	2	$3K = \frac{3}{10}$
	3	$5K = \frac{5}{10}$
	4	$8K = \frac{8}{10}$
	5	$8K + K^2 = \frac{81}{100}$
	6	$8K + 3K^2 = \frac{83}{100}$
	7	$9K + 10K^2 = 1$

$$\frac{4}{5} = 0.8$$

iii) $P(X \leq a) > \frac{1}{2}$

$$\Rightarrow a = 4$$

Distribution Function or cumulative Distribution Function (c.d.f.) -

Let x be a random variable. The funⁿ F defined for all real x by

$$F(x) = F_x(x) = P(X \leq x), -\infty < x < \infty$$

$$= \int_{-\infty}^x F(x) dx$$

is called the distribution funⁿ of the random variable x .

A distribution funⁿ is also called the cumulative distribution funⁿ.

$P(X \leq x)$ is the prob. that the random variable x (discrete or continuous) takes the values $\leq x$.

i) $0 \leq F(x) \leq 1 : -\infty < x < \infty$

$$dF(x) = F(x) dx$$

Prob. differential of x

a) A random variable x has the following (P.M.F) values of x

$$x = 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$P(X=x) = k \quad \frac{k}{12} \quad k \quad 2k - \frac{1}{5} \quad \frac{9}{120}$$

Find $P(X=2)$, $P(X \leq 1)$, $P(X < 4)$, $P(X > 3)$

~~Soln~~
we know that $\sum_{x=0}^4 P(X=x_i) = 1$

$$\Rightarrow P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$$

$$\Rightarrow K + \frac{K}{2} + K + 2K - \frac{1}{5} + \frac{9}{20} = 1$$

$$\text{solution} \Rightarrow 12.5K + 10K + 20K + 40K - 4 + 9 = 20$$

$\Rightarrow 20 \times (1+2+4+5) \text{ holds true}$

$$\Rightarrow 90K + 5 = 20$$

$$\Rightarrow 90K = 15$$

$$\Rightarrow K = \frac{1}{6}$$

$$\text{Now } P(X=2) = K = \frac{1}{6}$$

$$P(X \leq 1) = P(X=0) + P(X=1)$$

$$= K + \frac{K}{2}$$

$$= \frac{3K}{2} = \frac{3}{2} \times \frac{1}{6} = \frac{1}{4}$$

$$P(X < 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= K + \frac{K}{2} + K + 2K - \frac{1}{5}$$

$$= 10K + 5K + 10K + 20K - 2$$

$= 10$

$$= 45K - 2 = 45 \times \frac{1}{6} - 2$$

$= 10$

$= 15 - 4$

$$= \frac{11}{20}$$

$$(1-x)^2, (1-x) + (1-x) + (1-x) + (1-x) = 4(1-x)$$

$$P(X > 3) = P(X=4) = \frac{9}{20}$$

a) For what value of k $f(x) = k \left(\frac{1}{2}\right)^x$ will represent P.M.F of a random variable x . (10 M.A)

Soln we know that

$$\sum_{x=1}^{\infty} P(F(x)) = 1 \quad \text{P.M.F.}$$

$$\Rightarrow \sum_{x=1}^{\infty} k \left(\frac{1}{2}\right)^x = 1 \quad \text{Substituting}$$

$$\Rightarrow k \left\{ \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots \right\} = 1$$

$$\Rightarrow k \left[\frac{\frac{1}{3}}{1 - \left(\frac{1}{3}\right)} \right] = 1$$

$$\Rightarrow k \left(\frac{1}{2}\right) = 1$$

$$\Rightarrow k = 2$$

a) $F(x) = \frac{1}{2}(3-x)$, $0 < x < 1$

$$= \frac{1}{2}(3+x)$$
, $1 \leq x < 2$

Does $f(x)$ represent prob. density fun?

Soln

$$\int_0^1 \frac{1}{2}(3-x) dx + \int_1^2 \frac{1}{2}(3+x) dx$$

$$= \frac{1}{2} \left[3x - \frac{x^2}{2} \right]_0^1 + \frac{1}{2} \left[3x + \frac{x^2}{2} \right]_1^2$$

$$= \frac{1}{2} \left(3 - \frac{1}{2} \right) + \frac{1}{2} \left(6 + 2 - 3 - \frac{1}{2} \right)$$

$$= \frac{1}{2} \left\{ \frac{5}{2} + 5 - \frac{1}{2} \right\}$$

$$= \frac{1}{2} \left\{ \frac{5}{2} + \frac{9}{2} \right\} = \frac{7}{2} \neq 1$$

$\therefore f(x)$ does not represent p.d.f.

a) Let x be a continuous random variable with p.d.f

$$f(x) = \begin{cases} ax & 0 \leq x < 1 \\ a & 1 \leq x \leq 2 \\ -ax + 3a & 2 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

i) Determine the constant

ii) compute $P(x \leq 1.5)$

Soln

$$\int_0^2 ax dx + \int_1^2 a dx + \int_2^3 (-ax + 3a) dx = 1$$

$$= \left[\frac{ax^2}{2} \right]_0^2 + [ax]_1^2 + \left[-\frac{ax^2}{2} + 3ax \right]_2^3 = 1$$

$$= \frac{9}{2} + a - \frac{9}{2} + 9a + 2a - 6a = 1$$

$$\Rightarrow 6a - 4a = 1$$

$$\Rightarrow 2a = 1$$

$$\Rightarrow a = \frac{1}{2}$$

iii) $P(x \leq 1.5) = \int F(x) dx$

$$= \int_0^{1.5} F(x) dx + \int_1^{1.5} F(x) dx$$

$$= \int_0^{1.5} ax dx + \int_1^{1.5} a dx$$

$$= \left[\frac{ax^2}{2} \right]_0^{1.5} + [ax]_1^{1.5}$$

$$= \frac{9}{2} + 1.5a - a$$

$$= \frac{a+3a-2a}{2} = \frac{2a}{2}$$

$$\Rightarrow a = \underline{\underline{1}}$$

$$\Rightarrow a = \frac{1}{2}$$